[Total No. of Questions - 9] [Total No. of Printed Pages - 4]

Dec.-22-1355

CS-102L (Mathematical Concepts for Applied Intelligence)

B.Tech. 1st (CBCS)

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt Five questions in all, selecting one question each from sections A, B, C and D. Question no. 9 is compulsory.

SECTION - A

1. (a) Solve the equation:

$$7x - \frac{3}{4} = \frac{1}{5} - x \qquad (5)$$

(b) If α and β are the zeros of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, Find the values of k. (5)

2. (a) Determine two positive numbers whose sum is 300 and whose product is maximum. (5)

(b) Find a, b, c so that the function :

$$f(x) = \begin{cases} x^3 + 3x + 5, & x \le 1 \\ ax^2 + bx + c, & x > 1 \end{cases}$$

is twice differentiable at x = 1. (5)

SECTION - B

3. (a) For what value of λ the vectors:

$\hat{i} + 2\hat{j} + 3\hat{k}$, $\lambda\hat{i} - \hat{j} - \hat{k}$ and $3\hat{i} - 4\hat{j} + 3\hat{k}$ are coplanar? (5)

(b) Find a unit vector coplanar with $\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to $\hat{i} + \hat{j} + \hat{k}$. (5)

4. (a) Solve the following system of linear equations by the matrix method:

$2x + y - z = 6$

$3x - y + 2z = 3$

$x + 2y - z = 5$ (5)

(b) Find all the eigen values and eigen vectors of the following matrix:

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix} \qquad (5)$$

SECTION - C

5. (a) If F is the distributive function of the random variable X and if a < b, then prove that

$$P(a < X \le b) = F(b) - F(a) \qquad (5)$$

(b) State and Prove Baye's Theorem. (5)

6. (a) A random variable X has exponential distribution with parameter λ = 5. Find its p.d.f., mean, variance and moment generating function. (5)

(b) Find the probability distribution of the number of heads when three coins are tossed simultaneously. (5)

SECTION - D

7. (a) Describe Merits and Limitations of sampling. Also write a note on systematic sampling. (5)

[P.T.O.]

(b) Describe briefly the procedure of Testing Hypothesis. (5)

8. (a) Test the significance of the correlation r = 0.5 from a sample of size 18 against hypothesis correlation p=0–0.7. (5)

(b) (i) Write the properties of a Good Estimator.

(ii) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased. (5)

(Compulsory Question)

9. Answer all the questions :

(a) Find the monic quadratic equation with 3 and 5 as its roots.

(b) Simplify:

$\sqrt[4]{81x^8y^4}$

(c) Show that the following vectors are coplanar:

$\bar{a} = 10\hat{i} - 12\hat{j} - 4\bar{k}$, $\bar{b} = -16\hat{i} + 22\hat{j} - 2\hat{k}$ and

$\bar{c} = 2\hat{i} - 8\hat{j} + 16\hat{k}$

(d) Find the extreme values of :

$f(x) = x^4 - 2x^2 + 3$

(e) If A is an n-rowed non-singular matrix, then prove that $f(A^{-1}) = f(A)$.

(f) Find the characteristic equation and eigen values of the following matrix:

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

(g) Three coins are tossed once find the probability that at least one head and one tail occur.

(h) Define Moment Generating Function (m.g.f).

(i) Define Central Limit Theorem.

(j) What do you mean by classification of data? (10×2=20)